

This is a new topic in which the Anabelian Geometry approach will be defined and compared with other approaches that are distinct from it such as [non-Abelian algebraic topology](#) and [noncommutative geometry](#). The latter two areas have already made an impact on [quantum theories](#) that seek a new setting beyond SUSY—the Standard Model of modern physics. Moreover, it is also possible to consider in this topic novel, possible approaches to relativity theories, especially to [general relativity](#) on spacetimes that are more general than pseudo- or quasi- Riemannian ‘spaces’. Furthermore, other [theoretical physics](#) and physical mathematics developments may expand specific Anabelian Geometry applications to [quantum geometry](#) and [Quantum Algebraic Topology](#).

1 Anabelian Geometry

The area of mathematics called *Anabelian Geometry (AAG)* began with Alexander Grothendieck’s introduction of the term in his seminal and influential report “*Esquisse d’un Programme*” (1) produced in 1980. The basic setting of his anabelian geometry is that of the [algebraic fundamental group](#) $\mathcal{G}$ of an *algebraic variety* X (which is a basic [concept](#) in Algebraic Geometry), and also possibly a more generally defined, but related, geometric [object](#). The *algebraic fundamental group*, $\mathcal{G}$, in this case determines how the algebraic variety X can be mapped into, or linked to, another geometric object Y , assuming that $\mathcal{G}$ is [non-Abelian](#) or [noncommutative](#). This specific approach differs significantly, of course, from that of Noncommutative Geometry introduced by Alain Connes. It also differs from the main-stream Nonabelian Algebraic Topology (NAAT)’s generalized approach to topology in terms of [groupoids](#) and [fundamental groupoids](#) of a [topological](#) space (that generalize the concept of fundamental group), as well as from that of [higher dimensional algebra \(HDA\)](#). Thus, the fundamental anabelian question posed by Grothendieck was, and is: “*how much information about the isomorphism class of the variety X is contained in the knowledge of the étale fundamental group?*” (on p. 2 in [http : //www.math.jussieu.fr/ leila/SchnepsLM.pdf](http://www.math.jussieu.fr/~leila/SchnepsLM.pdf)). At this point, stepping down from the general, abstract setting of the Anabelian Geometry it would be useful to consider a specific, concrete example.

1.1 A Concrete Example

In the case of curves, C , these could be either *affine* (as in [Einstein’s](#) or Weyl’s approaches to General Relativity), or *projective*, as in a variety V . Consider here a specific hyperbolic curve H , that is defined as the complement of n points in a *projective algebraic curve of genus g* , which is assumed to be both smooth and irreducible, and also defined over a [field](#)

K (that is finitely generated over its *prime field*) such that:

$$2 - 2g - n < 0$$

. Grothendieck conjectured in 1979 that the *fundamental group* $\mathcal{G}$ of C , which is a *profinite group*, determines the curve C itself, or that the *isomorphism class* of $\mathcal{G}$ determines the isomorphism class of C ; this also points towards a conjecture regarding the *natural equivalence* η of their corresponding *categories*.

1.2 Generalizations

Much more elaborate, generalizations of Grothendieck's Anabelian Geometry are possible by considering higher-dimensional, *pro-l*, *Hom*-versions, and so on, involving for example fundamental groupoids and fundamental *double groupoids* (2).

1.3 References

1. [Alexander Grothendieck](#). 1984. "Esquisse d'un Programme", (1984 manuscript), published in "Geometric Galois Actions", L. Schneps, P. Lochak, eds., London Math. Soc. Lecture Notes 242, Cambridge University Press, 1997, pp.3–48; English transl., *ibid.*, pp. 243–283.
2. Jochen Koenigsmann. 2001. Anabelian geometry over almost arbitrary fields. [http : //www.uni - math.gwdg.de/tschinkel/SS04/koenigsmann2.pdf](http://www.uni-math.gwdg.de/tschinkel/SS04/koenigsmann2.pdf)
3. S. Mochizuki, H. Nakamura, A. Tamagawa. "The Grothendieck Conjecture on the fundamental groups of algebraic curves", *Sugaku Expositions*, **14**(1), (2001), 31–53.

[...work](#) in progress